

# Photon electroproduction off nuclei in the $\Delta$ -resonance region

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## Abstract

The cross section for the  $A(e, e'\gamma)A$  reaction is calculated, investigating the contribution from the nuclear target with respect to the radiative corrections from the electron. The reaction mechanism is studied for photon emission in the  $\Delta$ -resonance region, varying the scattering geometry and analyzing the most favourable kinematical conditions to extract information on the nuclear system.

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# 1 Introduction

The photon electroproduction process off nuclei is potentially a very useful tool to investigate the nuclear structure. In the one photon exchange approximation, the reaction is described by the coherent sum of the Bethe-Heitler (BH) amplitude and the full virtual Compton scattering (FVCS) amplitude. The first process describes the emission of bremsstrahlung photons by the electron in the nuclear electromagnetic field and is exactly calculable from QED. The second process corresponds to electron scattering by exchange of a virtual photon which is scattered by the nucleus into a real final photon and is given by a linear combination of virtual Compton scattering (VCS) amplitudes. The competition between the two processes makes a very difficult task to extract experimentally the interesting information on the nuclear response involved in the reaction. Such experiments have been proposed in the past in the energy range of the giant resonances [1, 2], exploring the best kinematical conditions to extract information on the first excited nuclear levels. VCS in the same energy region has been also considered in ref. [3] within a new formulation in terms of nuclear generalized polarizabilities and an experiment has been performed to study the 4,44 MeV excited state ( $J^\pi = 2^+$ ) in  $^{12}\text{C}$  [4].

Recently, new interest has emerged to study the nucleon structure through VCS [5, 6]. Experiments have already been scheduled at MAMI-B [7] below the pion production threshold in order to explore the non perturbative structure of the nucleon, while proposed experiments at CEBAF [8] will extend the measurements to higher virtual photon momenta.

In this paper, we propose to investigate the same reaction mechanism in the case of a nuclear target, discussing the possibility to extract relevant information on the nuclear structure at intermediate energies. VCS in the  $\Delta$ -resonance region has already been discussed in ref. [9], focusing the attention on the new accessible information with respect to the real Compton scattering. The discussion is now extended to include the evaluation of photon bremsstrahlung contribution of the electrons, exploring the interplay between the BH and the FVCS amplitude in different scattering geometries and trying to disentangle as clearly as possible the pure nuclear contribution.

In sect. 2, the explicit expression for the cross section of the  $A(e, e'\gamma)A$  reaction is given, separating the contribution from the BH, the FVCS and the interference terms. Results for  $^4\text{He}$  at the electron energies accessible

at MAMI and CEBAF are presented in sect. 3 and concluding remarks are reported in the final section.

## 2 Cross section of photon electroproduction off nuclei

To describe the photon electroproduction reaction off nuclei

$$e + A \rightarrow e' + A + \gamma, \quad (1)$$

$$(h, k) \quad (p) \quad (h', k') \quad (p') \quad (\varepsilon_{\lambda'}^\mu, q'),$$

we choose to work in the laboratory frame system and denote with  $h$ ,  $h'$  and  $k^\mu = (E, \vec{k})$ ,  $k'^\mu = (E', \vec{k}')$  the helicity and the four-momentum of the initial and final electron, respectively. Neglecting the nucleus recoil, the four-momentum of the nucleus in the initial and final state is  $p^\mu = p'^\mu = (M_A, 0)$ , while  $\varepsilon_{\lambda'}^\mu(q'^\mu)$  and  $q'^\mu = (\omega' = q', \vec{q}')$  are the polarization vector and four-momentum of the final photon, respectively.

Without observing polarization effects, the differential cross section is given by

$$d\sigma \equiv \frac{d^5\sigma}{d\Omega_{e'} d\Omega_\gamma d\omega'} = \frac{m_e^2}{(2\pi)^5} \frac{\omega'}{2} \frac{|\vec{k}'|}{|\vec{k}|} \sum_{h', \lambda'} \overline{\sum_h} |M_{h' \lambda', h}^{\text{BH}} + M_{h' \lambda', h}^{\text{FVCS}}|^2, \quad (2)$$

where  $m_e$  is the electron mass and  $M_{h' \lambda', h}^{\text{BH}}$  and  $M_{h' \lambda', h}^{\text{FVCS}}$  are the helicity amplitudes for the BH and FVCS processes, respectively. Due to the coherence of the two effects, the cross section is the sum of three terms

$$d\sigma = d\sigma^{\text{BH}} + d\sigma^{\text{FVCS}} + d\sigma^{\text{INT}}, \quad (3)$$

where  $d\sigma^{\text{BH}}$  and  $d\sigma^{\text{FVCS}}$  are the contributions of photon emission by the electron and the nucleus, while  $d\sigma^{\text{INT}}$  is obtained from the interference between the BH and FVCS amplitudes.

The BH amplitude is evaluated in the external field approximation, where the nucleus is treated as source of a static external Coulomb field, which transfers to the electron the momentum  $\vec{\kappa} = \vec{p}' + \vec{q}' - \vec{p}$ . The explicit expression for the BH amplitude reads

$$M_{h' \lambda', h}^{\text{BH}} = -i e^2 A^\mu(\vec{\kappa}) j_{\mu\nu}^{\text{BH}}(h, h') \varepsilon_{\lambda'}^\nu(q'^\mu), \quad (4)$$

where  $A^\mu(\vec{\kappa}) = (F(\vec{\kappa})/|\vec{\kappa}|^2, 0, 0, 0)$  is the momentum-space Coulomb potential corresponding to the nuclear charge form factor  $F(\vec{\kappa})$  and  $j_{\mu\nu}^{BH}(h, h')$  is the leptonic current for the process [10]. Finally, from the squared modulus of the BH amplitude the well known Bethe-Heitler formula for  $d\sigma^{BH}$  [10] is derived.

In the one photon exchange approximation, the FVCS amplitude is given by a linear combination of VCS amplitudes

$$M_{h'\lambda',h}^{\text{FVCS}} = \frac{ie}{Q^2} \sum_{\lambda} (-1)^{\lambda} \varepsilon_{\lambda}^{\mu*}(q) j_{\mu}(h, h') M_{\lambda'\lambda}^{\text{VCS}}, \quad (5)$$

where  $j_{\mu}(h, h')$  is the electron current,  $\varepsilon_{\lambda}^{\mu}(q^{\mu})$  and  $q^{\mu} = k^{\mu} - k'^{\mu} = (\omega, \vec{q})$  are the polarization vector and four-momentum of the virtual intermediate photon, respectively and  $Q^2 = q^2 - \omega^2$ .

$M_{\lambda'\lambda}^{\text{VCS}}$  is the model-dependent nuclear transition amplitude, describing the scattering off nucleus of a virtual photon with helicity  $\lambda = 0 \pm 1$  into a real photon with helicity  $\lambda' = \pm 1$ . At intermediate energies, this term is dominated by the resonant contribution describing the excitation of the  $\Delta(1232)$  isobar inside the nucleus. As explained in details in ref. [9, 12], the  $\Delta$ -resonance contribution can be satisfactorily evaluated within a local-density approximation to the  $\Delta$ -hole model. A background non-resonant contribution can be included, taking into account the seagull diagram and the scattering amplitude due to s-wave pion production and absorption on a single nucleon. Within the framework of this model, the FVCS cross section is directly calculated from the expression

$$d\sigma^{\text{FVCS}} = \frac{e^2}{(2\pi)^5} \frac{\omega'}{8Q^4} \frac{|\vec{k}'|}{|\vec{k}|} \sum_{\lambda, \lambda', \bar{\lambda}} (M_{\lambda'\lambda}^{\text{VCS}}) L_{\lambda\bar{\lambda}} (M_{\lambda'\bar{\lambda}}^{\text{VCS}})^*, \quad (6)$$

where  $L_{\lambda\bar{\lambda}}$  is the lepton tensor (see, e.g., ref. [11]).

The summation in eq. (6) can be explicitly written in terms of four structure functions of the nucleus

$$\begin{aligned} \sum_{\lambda, \lambda', \bar{\lambda}} (M_{\lambda'\lambda}^{\text{VCS}}) L_{\lambda\bar{\lambda}} (M_{\lambda'\bar{\lambda}}^{\text{VCS}})^* = \\ = L_{00} W_L + L_{11} 2W_T + L_{01} W_{LT} \cos \alpha + L_{1-1} 2W_{TT} \cos 2\alpha, \end{aligned} \quad (7)$$

where  $\alpha$  is the azimuthal angle of the emitted photon with respect to the electron scattering plane. The pure transverse structure function  $W_T$  is the

incoherent sum of photon-helicity flip and non-flip contributions, while the transverse-transverse  $W_{\text{TT}}$  gives the interference contribution from helicity flip and non-flip amplitudes. The pure longitudinal response is given by the structure function  $W_L$  and longitudinal-transverse interference contributions are contained in  $W_{\text{LT}}$ . With respect to the real Compton scattering, new information are available through the  $W_L$  and  $W_{\text{LT}}$  responses, while the same  $W_T$  and  $W_{\text{TT}}$  structure functions can now be explored varying independently the energy and momentum of the incoming photon.

The VCS amplitudes come into play also to determine the interference contribution  $d\sigma^{\text{INT}}$ . Separating the leptonic and nuclear terms, one has

$$d\sigma^{\text{INT}} = \frac{m_e^2 \omega'}{(2\pi)^5} \frac{|\vec{k}'|}{|\vec{k}|} \frac{F(\vec{\kappa}) e^3}{|\vec{\kappa}|^2 Q^2} \text{Re} \left\{ \sum_{\lambda, \lambda'} L_{\lambda\lambda'}^{\text{INT}} (M_{\lambda'\lambda}^{\text{VCS}})^* \right\}, \quad (8)$$

where now the helicity amplitudes  $M_{\lambda'\lambda}^{\text{VCS}}$  are linearly combined by the tensor  $L_{\lambda\lambda'}^{\text{INT}}$ , derived by the interference between the leptonic currents in the BH and FVCS amplitude.

### 3 Results

The total information on the nuclear dynamics is summarized in the nuclear structure functions of eq. (6). To access experimentally these responses, we face the problem to find those kinematical conditions where the bremsstrahlung contributions of the electron are as small as possible. The relevant variables to determine the scattering geometry are given by

$$(E, E', \theta_{e'}, q, \omega, \theta_\gamma, \alpha), \quad (9)$$

where  $\theta_{e'}$  is the scattering angle of the outgoing electron with respect to the initial electron and  $\theta_\gamma$  is the scattering angle of the final photon with respect to the virtual one. Neglecting the recoil of the nucleus, the nuclear structure functions depend only on the variables  $\omega = \omega'$ ,  $\theta_\gamma$  and  $q$ . Since we are interested in the nuclear dynamics in the  $\Delta$ -resonance region, we keep fixed  $\omega = 310$  MeV and explore the nuclear responses as a function of  $\theta_\gamma$  in different regions of  $q$ . For given values of  $q$  and  $\omega$ , the electron variables  $E, E'$  and  $\theta_{e'}$  are related by two conditions

$$E - E' = \omega, \quad (10)$$

$$|\vec{q}^2| = E^2 + E'^2 - 2EE' \cos \theta_{e'}. \quad (11)$$

As a consequence, we have at our disposal only one independent variable, which we choose to be the incoming electron energy  $E$ .

The azimuthal angle  $\alpha$  of the outgoing photon is chosen in a such way to separate the different contributions of the structure functions in the total cross section. Measurements of the cross section at complementary angles  $\alpha$  can be combined to define the two quantities

$$\begin{aligned} A_+(\alpha) &= d\sigma(\alpha) + d\sigma(180^\circ - \alpha), \\ A_-(\alpha) &= d\sigma(\alpha) - d\sigma(180^\circ - \alpha). \end{aligned} \quad (12)$$

From eq. (6), we deduce that in the left-right asymmetry  $A_-$  the  $W_{LT}$  contribution to the FVCS cross section is singled out. Since out-of-plane kinematics is in general favoured to reduce the BH contaminations [7, 8], we will take the two values  $\alpha = 45^\circ$  and  $\alpha = 135^\circ$ . On the other hand, these conditions allow to estimate the relative contribution of the  $W_L$  and  $W_T$  structure functions in the  $A_+$  combination. The importance of the pure longitudinal response is completely negligible with respect to  $W_T$  [9] and in the regions of small BH contaminations we could extract from the  $A_+$  measurement direct information on the  $W_T$  structure function.

A different analysis has been recently performed in ref. [13], where the interplay between the BH and FVCS contributions is investigated in the  $\Delta$ -resonance region at different values of  $E$  and  $\theta_{e'}$  and integrating the cross section over the photon azimuthal angle. With the choice of  $(E, q, \alpha)$  as the set of independent variables, we can now examine more closely the role of the nuclear responses, emphasizing the new information available with respect to real Compton scattering from the behaviour as a function of the momentum transfer  $q$ .

In figs. 1-3, results for  $A_+(45^\circ)$  are shown as a function of the photon scattering angle  $\theta_\gamma$ , separating the BH and FVCS contributions with dotted and dot-dashed lines, respectively. The calculations are performed for  $^4\text{He}$  at three different values of the incoming electron energy ( $E = 500, 885$  and  $2000$  MeV), keeping fixed  $\omega = 310$  MeV and investigating the region of the virtual photon momentum  $q$  between 330 and 480 MeV. BH contaminations are dominant at forward angles of the outgoing photon, becoming negligible in the backward region. In particular, keeping the electron energy fixed at

$E = 500$  MeV, the nuclear contribution can be better seen at small values of  $q$ , where it becomes the leading term for  $\theta_\gamma \geq 60^\circ$ . On the other hand, increasing the electron energy, we observe the same trend as a function of  $q$ , while the angular region where FVCS dominates is extended to smaller  $\theta_\gamma$  (for instance,  $\theta_\gamma \geq 40^\circ$  at  $E = 2000$  MeV and  $q = 330$  MeV).

Due to the angular dependence on  $(1 - \cos\theta_\gamma)$  and  $(1 + \cos\theta_\gamma)$  of the helicity flip and non-flip amplitudes, respectively, the  $W_T$  structure function is dominated at large angle by the helicity flip contribution. This term is not yet well understood on the theoretical point of view [12, 14] and important information in a large range of  $q$  could be clearly extracted from VCS experiments.

Results for the left-right asymmetry  $A_-(\alpha = 45^\circ)$  are plotted in figs. 4-6 for the same kinematics previously explained. The  $W_{LT}$  contributions are given in absolute value and the minima correspond to a sign change from positive to negative values. As expected, the small longitudinal term is very hard to extract, even if the overwhelming BH contribution falls down in the backward scattering region.

The most interesting situation to investigate is at the electron energy  $E = 2000$  MeV. In the  $q$  range between 330 and 430 MeV and at large scattering angles, the BH term becomes negligible or is completely canceled by the negative  $W_{LT}$  contribution and the left-right asymmetry is controlled by the positive interference between the FVCS and the BH amplitudes. On the other hand, in the interference cross section  $d\sigma^{\text{INT}}$  the VCS amplitudes are linearly summed, with a negligible importance of the longitudinal part with respect to the transverse one. At the highest  $q = 480$  MeV, the BH contribution is the only important term and becomes dominant even at smaller values of  $q$  for decreasing electron energy.

More complex combinations of the total cross section can be analyzed to extract also the  $W_{TT}$  contribution (for instance, by subtracting twice the cross section at  $\alpha = 90^\circ$  from  $A_+(45^\circ)$ ), but the small values of the involved cross sections should require too high-precision experiments.

## 4 Concluding remarks

The cross section for the  $A(e, e'\gamma)A$  reaction has been calculated in the  $\Delta$ -resonance region, investigating the interplay between the BH and FVCS am-

plitudes for various scattering geometries. Out-of-plane kinematics, with  $\alpha = 45^\circ$  and  $\alpha = 135^\circ$ , has been chosen to disentangle the role of the  $W_{LT}$  and  $W_T$  structure functions in the plus and minus combinations of the total cross sections.

In order to explore the nuclear responses as a function of the momentum  $q$  in the range between 330 and 480 MeV, the most favourable case occurs for high energy of the incoming electron. The pure transverse  $W_T$  can be clearly separated from BH contaminations at backward scattering angles, where new interesting information could be obtained on the helicity flip amplitude contributions.

More difficult appears to explore the longitudinal nuclear response. On one hand, in the  $W_{LT}$  structure function the small longitudinal contribution is amplified by the interference with the transverse term, but we have to compete with the overwhelming BH contaminations in the measurement of the left-right asymmetry. On the other hand, in the regions where the BH and FVCS interference is pronounced, the role of the longitudinal amplitude in  $d\sigma^{\text{INT}}$  is obscured by the transverse terms.

## References

- [1] D. F. Hubbard and M. E. Rose, Nucl. Phys. 84 (1966) 337.
- [2] H. L. Acker and M. E. Rose, Ann of Phys. 44 (1961) 336.
- [3] H. Arenhövel and D. Drechsel, Nucl. Phys. A233 (1974) 153.
- [4] C. N. Papanicolas et al., Phys. Rev. Lett. 54 (1985) 26.
- [5] G.R. Farrar and H. Zhang, Phys. Rev. D41 (1990) 3348.
- [6] P.A.M. Guichon, G.O. Liu, and A.W. Thomas, Nucl. Phys. A591 (1995) 606.
- [7] G. Audit et al., MAMI proposal MAMI proposal A1/1-95 (1995).
- [8] G. Audit et al., CEBAF proposal PR-93-050 (1993).
- [9] B. Pasquini and S. Boffi, preprint nucl-th/9605003 and to be published in Phys. Lett. B.



- [10] See for instance, J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons*, 2nd edn., Springer, New York, 1976, section 15-6.
- [11] S. Boffi, C. Giusti and F.D. Pacati, Phys. Rep. 226 (1993) 1.
- [12] B. Pasquini and S. Boffi, Nucl. Phys. A598 (1996) 485.
- [13] A. Gil, J. A. Gomez Tejedor and E. Oset, preprint nucl-th/9608004.
- [14] J.H. Koch, E.J. Moniz and N. Ohtsuka, Ann. Phys. (NY) 154 (1984) 99.

### Figure captions

Fig. 1.  $A_+$  combination of the differential cross section for photon electroproduction off  $^4\text{He}$ , calculated at  $\alpha = 45^\circ$  as a function of the photon scattering angle  $\theta_\gamma$  for the incoming electron energy  $E = 500$  MeV. The transfer energy  $E - E'$  is fixed at 310 MeV and the virtual photon momentum is taken at the different values  $q = 330, 380, 430$  and  $480$  MeV. The dashed and dot-dashed lines are the separate contributions of the BH and FVCS processes, respectively, while the solid line is obtained from the coherent sum of the two contributions.

Fig. 2. The same as in fig. 1 but for  $E = 885$  MeV.

Fig. 3. The same as in fig. 1 but for  $E = 2000$  MeV.

Fig. 4.  $A_-$  asymmetry calculated in the same kinematical conditions as in fig. 1.

Fig. 5. The same as in fig. 4 but for  $E = 885$  MeV.

Fig. 6. The same as in fig. 4 but for  $E = 2000$  MeV.

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